Confined Pressuremeter Tests for the Assessment of the Theoretically Back-Calculated Cross-Anisotropic Elastic Moduli
Adolfo Foriero and Ferdinand Ciza

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Reference

ABSTRACT
This study considered the determination of the elastic cross-anisotropic moduli of two base course materials. The cross-anisotropic moduli were back-calculated from a series of vertical and horizontal laboratory pressuremeter tests in a confined mold. First, a theoretical model of an expanding cylindrical cavity in an elastic cross-anisotropic confined medium was developed. Subsequently, an estimate of four \(G_{hh}, G_{0hh}, \mu_{vh}, \text{ and } \mu_{hv}\) of the five independent cross-anisotropic moduli were determined based on a fixed point iteration scheme, which processed the analytically back-calculated test results. Finally, in order to assess the estimated moduli, results of the finite element simulations of the pressuremeter lab tests were compared with the actual measurements.

Keywords
pressuremeter, cross-anisotropic, confined cavity, analytic model, finite element

Introduction
The pressuremeter has been mainly utilized in foundation engineering. However, its use for obtaining parameters in pavement design via the PENCEL pressuremeter is quite known and ongoing (Cosentino and Briaud 1989; Farid et al. 2013). The philosophy of design for pavements is different from that of foundations. The design of pavements is governed by very small tolerable deflections because, for safety reasons, the serviceability of traffic is the primary goal. Consequently, the factor of safety of pavements against bearing capacity failure is quite substantial when compared to the classic factor safety of 1.5 to 3 used in foundation design.
To date, most of the PENCEL pressuremeter tests were carried out in the subgrade of a typical pavement cross section (Briaud and Shields 1979; Cosentino and Briaud 1989; Farid et al. 2013). Generally the parameters are back-calculated from the pressuremeter curve by assuming the subgrade medium as isotropic. If the subgrade is composed mainly of coarse graded materials, then the parameters obtained are the shear modulus, in situ total horizontal stress, and peak friction angle. For a fine subgrade, one obtains, in addition to the already mentioned parameters, the undrained shear strength and the coefficient of horizontal consolidation.

In order to predict realistic surface deformations, a proper choice of constitutive model is necessary throughout the pavement cross section. Specifically, accurate estimates of deformations in stiff soils require, at the very least, the use of models depicting the soil as linearly elastic cross-anisotropic. This is particularly evident beneath a pavement where inherent and induced anisotropy is likely to be present.

In this study, emphasis was placed on the base course material of a typical pavement cross section. This necessitated the development of a new method, which determines the cross-anisotropic moduli using the PENCEL pressuremeter in a confined laboratory test. The reason for this approach is twofold: (1) the standard PENCEL pressuremeter has an effective length that is greater than that of a typical base course layer, and (2) the cross-anisotropic elastic moduli are pressure dependent in a confined test.

Subsequently, a theory was developed in order to extract or back-calculate the cross-anisotropic elastic moduli using results obtained from the vertical setup of the pressuremeter test. The moduli were then incorporated in the finite element commercial software COMSOL in order to model a vertically and horizontally pressurized cavity in a confined mold. Results of the finite element simulations were also compared to laboratory tests where effectively the PENCEL pressuremeter was laid horizontally in the same confined mold as the vertical pressuremeter tests.

Origin and Properties of Material Used in the Experimental Program

All tests realized in this study were based on unbounded granular material commonly identified by the Québec office of normalization (Government of Québec 2010). Samples were collected from two sites in order to cover representative soils used as base course material in the Québec context. The materials tested were both obtained from soil deposits in Québec city. They are respectively sand (identified as MG112 material) from La Sablière Robitaille and limestone gravel (identified as MG20 gravel) from Carrière Union.

Sieve analyses were conducted on these materials and produced the granulometric curves given in Fig. 1.

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**FIG. 1** Grain size distribution curves of materials used in the pressuremeter tests.

**FIG. 2** Pressuremeter apparatus.
The pressuremeter equipment used in this study (Fig. 2) was distributed by Roctest (Roctest Inc. 2005) and consists specifically of the following:

1. The PENCEL probe, a cylindrical hollow body with an inflatable metallic sheath. The sheath is held in place by two tapered metal rings and two lock nuts. The probe is fitted with a quick disconnect at its upper extremity and a saturation plug at its lower extremity. The male quick connect accepts the tubing leading from the pressure-volume control unit. In the present investigation, the length and diameter of the probe are, respectively, 240 and 32 mm.

2. The Texam control unit, made up of a metal case that houses the main cylinder, with four quick connectors and the control valve. A manual actuator operates the piston and a digital pressure gauge. The control unit has a number of important features. Among these features is a precise command of the imposed strain and, consequently, the possibility to generate an unlimited number of test points. Recycling is another asset because of the probe’s deflationary properties and leads to the execution of load cycles with very little effort. Another advantage is the overall operation of the control unit, which is quite flexible and simple.

3. The tubing, a single conduit fitted with shut-off quick connectors to maintain saturation. This allows water to be directed from the control unit to the probe and thereby inflate the inner rubber membrane. The valve also allows the system to remain saturated when the tubing is detached from the control unit.

COMPACTION MOLD

Figs. 3 to 5 illustrate the prototype compaction mold designed at Laval University. The cylindrical mold measures 335 mm in height and has an interior and exterior diameter, respectively, of 330 and 350 mm. The dimensions of the mold were based primarily on those of the PENCEL pressuremeter. In particular, a cylindrical tube with the same external diameter (32 mm) as that of the PENCEL pressuremeter is placed at the center of the mold. This facilitates compaction of the surrounding material. As shown in Fig. 3, a cylindrical tube, during the compaction phase, is held in place by a rotating tripod, which caps the cylinder. The rotation of the tripod is necessary in order to ensure uniform compaction of the surrounding confined material.

After the compaction phase, the tripod and the cylindrical tube are removed, and the probe is inserted in the cavity.

FIG. 3 Compaction of the sub-base material in preparation for pressuremeter testing in a vertical cavity.

FIG. 4 Diagram of the designed prototype mold destined for vertical pressuremeter testing.

FIG. 5 Diagram of the designed prototype mold destined for horizontal pressuremeter testing.
mold is then capped with a metal plate having an opening of the same diameter as that of the probe. For the vertical tests, the mold cap is slid over the probe. This not only confines the soil but secures the vertical alignment of the probe (Fig. 4). For the horizontal tests, the probe is inserted through two circular openings in the mold wall that are oriented at diametrically opposite ends. After the compaction phase, the mold is capped (Fig. 5).

Figs. 6 and 7 illustrate the actual vertical and horizontal pressuremeter tests based on the prototype design adapted for the laboratory facilities at Laval University.

DESCRIPTION OF LABORATORY PRESSUREMETER TESTS

The base course materials were compacted in the test mold at their respective optimum moisture content in order to enhance compaction efficiency. The values of the optimum moisture content and maximum dry unit weight were obtained with the standard Proctor test. The optimum moisture content and maximum dry unit weight for MG112 sand and MG20 gravel are, respectively, $c_d = 18.94 \text{kN/m}^3$, $\omega_{opt} = 11.18\%$ and $c_d = 21.72 \text{kN/m}^3$, $\omega_{opt} = 6.64\%$.

The compaction process was accomplished by consecutively placing seven quasi-uniform layers in the mold. The height of the first layer was ±65 mm while those of the last six layers were ±45 mm. Each layer was subjected to compaction using a vibrating hammer having a flat face as previously depicted in Fig. 3.

For this study, the basic working procedures of the pressuremeter lab tests are as follows: During tests, the wheel of the hand pump in the control unit is advanced in order to force water into the probe. As previously mentioned, the probe is installed in a cylindrical soil cavity of the rigid steel mold. As a result of the resistance offered by the soil, both the pressure and volume of injected water are read, respectively, on the gage and counter of the control unit. For the unloading/reloading phase of the tests, the pressure and volume are decreased by reversing the rotation of the hand pump wheel.

RESULTS OF LABORATORY-CONFINED PRESSUREMETER TESTS

As shown in Figs. 8 to 15, all pressuremeter tests were subjected to four unload-reload cycles. The primary reason for this is the almost inevitable disturbance of the soil around the cavity during the installation process, which makes the values of the initial portion of the pressuremeter curve extremely unreliable. Moreover, the confined pressuremeter theory, developed in the next section of this paper, requires at least four unload-reload cycles in order to estimate four ($G_{h0}$, $\mu_{h0}$, $\mu_{h0}$, and $\mu_{h0}$) of the five independent elastic cross-anisotropic parameters. The fifth elastic parameter $G_{h0}$ is obtained using a relationship proposed by Gazetas (1981) and is also covered in the next section of this paper.

The general trend of the test results indicate that the slopes of unload-reload cycles tend to increase as the cavity pressure increases. This behavior is observed regardless of the material being tested or the orientation of the cavity pressure (MG112 sand, Figs. 8 to 11; MG20 gravel, Figs. 12 to 15). Further evidence is that the cavity pressure at a given radial strain is generally higher for the MG20 gravel than for the MG112 sand.

Only vertical tests were utilized in the determination of the cross-anisotropic elastic parameters. The reason is that the analytic theory developed to back-calculate the cross-anisotropic moduli rests on a particular physical model, specifically, that of a confined cylindrical geometry subject to axisymmetric loading. For horizontal tests, the loading is nonsymmetric with respect to this same geometry. Furthermore, in the geotechnical field, pressuremeter testing is generally carried out in a vertical cavity. Consequently, the results of horizontal tests were utilized only for verification purposes in the ensuing finite element analysis. Table 1 shows the calculated slopes of the unload-reload
curves for the vertical cavity tests. In particular, the unload-reload slope \( \frac{dP}{dr} \) (where \( P \) is the cavity pressure and \( r \) is the radius of the probe) is required as an input to the theoretical model developed in the following section.

Theoretical Evaluation of the Cross-Anisotropic Elastic Moduli Based on a Confined Pressuremeter Test

The theoretical determination of the cross-anisotropic elastic moduli is strongly dependent upon the kinematic conditions imposed by the model. Here, one considers the problem of an expanding cylindrical vertical cavity in a confined mold. In this instance, the soil (contained in a rigid cylindrical mold) is also restrained from movement in the vertical direction by a steel plate that is bolted to the mold and, therefore, caps the soil (Fig. 4). In the horizontal direction, at a distance \( b \) from the center of the mold (corresponding to the interior radius of the mold), displacements are prevented by the sheer rigidity of the steel mold. Consequently, in the ensuing analysis, a typical lab test is modeled as a cylindrical cavity expansion with the PENCEL pressuremeter supplying the inner cavity pressure.

The equilibrium equation for a cylindrical cavity expansion \((\epsilon_z = 0, \sigma_z \neq 0)\) is generally expressed as

\[
\frac{d\sigma_r}{dr} + \frac{1}{r} (\sigma_r - \sigma_\theta) = 0
\]
where: \( \sigma_r, \sigma_{th} \) and \( r \) = the radial stress, the tangential stress, and the radial coordinate, respectively (Ladanyi and Foriero 1998; Hughes et al. 1977; Gibson and Anderson 1961).

For a linear elastic cross-anisotropic soil, the constitutive equations are given as

\[
\epsilon_r = \frac{\sigma_r}{E_h} - \frac{\mu_{th} \sigma_\theta}{E_h} - \frac{\mu_{th} \sigma_z}{E_v}
\]

(2)

\[
\epsilon_\theta = \frac{\sigma_\theta}{E_h} - \frac{\mu_{th} \sigma_r}{E_h} - \frac{\mu_{th} \sigma_z}{E_v}
\]

(3)

\[
\epsilon_z = \frac{\sigma_z}{E_v} - \frac{\mu_{th} \sigma_r}{E_h} - \frac{\mu_{th} \sigma_\theta}{E_h}
\]

(4)

where: \( \epsilon_r, \epsilon_\theta, \) and \( \epsilon_z \) = the strains in the radial, tangential, and vertical direction, respectively, and elastic parameters \( E_h, E_v, \mu_{th}, \mu_{th}, \) and \( \mu_{th} = \) the Young’s modulus in the horizontal (\( h \)) direction, the Young’s modulus in the vertical (\( z \)) direction, the Poisson’s ratio of strain in the horizontal direction caused by strain in the horizontal direction, the Poisson’s ratio of strain in the vertical direction caused by strain in the horizontal direction, and the Poisson’s ratio of strain in the horizontal direction caused by strain in the vertical direction, respectively.

By considering small strains and expressing, respectively, the radial and tangential strains as
\[ \epsilon_r = -\frac{du}{dr} \]  
and
\[ \epsilon_\theta = -\frac{u}{r} \]  

one obtains a sufficient number of equations (Eqs 1–6) to solve for the six unknowns \( u, \epsilon_r, \epsilon_\theta, \sigma_r, \sigma_\theta, \) and \( \sigma_z \). In the process of solving for these unknowns, Warren (1982) obtains the governing ordinary differential equation

\[ r^2 \frac{d^2 u}{dr^2} + r \frac{du}{dr} - u = 0 \]  

where:

\( u \) = the radial displacement.

This equation is similar to that obtained by Gibson and Anderson (1961) for the isotropic case.

The abovementioned previous authors solved this differential equation by considering a semi-infinite domain \( (r \to \infty) \). However, a different approach is followed hereafter because, as previously mentioned, the kinematics boundary conditions are dictated by the physical model. For the lab tests conducted in this study, the domain is of finite extent and the boundary conditions of the problem are given as

\[ r = a, \quad u = u_a \]  
and
\[ r = b, \quad u = 0 \]  

where:

\( a \) and \( b \) = the inner and outer radii of the cylindrical soil medium.

Physically, the inner radius \( a \) corresponds to the initial position of the PENCEL pressuremeter’s membrane while the outer radius \( b \) is the inner radial dimension of the mold.

The solution of Eq 7 subject to the boundary conditions in Eqs 8 and 9 yields for the radial displacements over the finite domain \( a \leq r \leq b \)

\[ u(r) = \frac{a}{a^2 - b^2} \left[ \frac{b^2}{r} - \frac{r^2}{b^2} \right] \]  

which in turn, via Eqs 5 and 6, leads to expressions for the strains

\[ \epsilon_r = -\frac{a}{b^2 - a^2} \left[ 1 + \frac{r^2}{b^2} \right] \]  
and
\[ \epsilon_\theta = -\frac{b^2}{a^2 - b^2} \left[ 1 - \frac{b^2}{r^2} \right] \]

An interesting result occurs here, particularly that the volumetric strain \( \epsilon_v = \epsilon_r + \epsilon_\theta + \epsilon_z \) is nonzero and constant as expressed by

\[ \epsilon_v = \frac{2a u_a}{b^3 - a^3} \]  

marking a major difference with the solution over a semi-infinite domain. The generalized shear strain

\[ \epsilon_s = \frac{2a u_a}{3(a^2 - b^2)r^2} \sqrt{3b^4 + r^4} \]  

The stress field is based on symmetry of the compliance matrix (deduced from Eqs 2–4), specifically that

\[ \frac{\mu_{hh}}{E_v} = \frac{\mu_{hv}}{E_h} \]  

while isotropy in the horizontal direction yields

\[ G_{hh} = \frac{E_v}{2(1 + \mu_{hh})} \]  

This gives for the radial, tangential, and vertical stress, respectively,

\[ \sigma_r = \frac{2G_{hh} a u_a}{(a^2 - b^2)(\mu_{hh} + 2\mu_{hh}\mu_{hv} - 1)} \]  
\[ \times \left[ 1 + \mu_{hh} + (1 + \mu_{hh} + 2\mu_{hh}\mu_{hv} - 1) \frac{b^2}{r^2} \right] \]  

\[ \sigma_\theta = \frac{2G_{hh} a u_a}{(a^2 - b^2)(\mu_{hh} + 2\mu_{hh}\mu_{hv} - 1)} \]  
\[ \times \left[ 1 + \mu_{hh} + (1 + \mu_{hh} + 2\mu_{hh}\mu_{hv} - 1) \frac{b^2}{r^2} \right] \]  

and

\[ \sigma_z = \frac{4E_v}{E_h} \frac{G_{hh}(\mu_{hh} + 1)\mu_{hh} u_a}{(a^2 - b^2)(\mu_{hh} + 2\mu_{hh}\mu_{hv} - 1)} \]  
\[ \times \left[ 1 + \mu_{hh} + (1 + \mu_{hh} + 2\mu_{hh}\mu_{hv} - 1) \frac{b^2}{r^2} \right] \]

Again, the marked departure from the solution obtained over a semi-infinite domain is confirmed by the mean stress derived as

\[ p = \frac{1}{3} (\sigma_r + \sigma_\theta + \sigma_z) \]  
\[ = \frac{4}{3(a^2 - b^2)(\mu_{hh} + 2\mu_{hh}\mu_{hv} - 1)} \left[ 1 + \mu_{hh} + \mu_{hv}\frac{E_v}{E_h} (\mu_{hh} + 1) \right] \]

Equations 13 and 20 confirm the fact that the pressuremeter does not load the surrounding soil in pure shear when the surrounding soil is confined (i.e., finite domain).

A practical result arises if one assumes that the radial stress (Eq 17) is equivalent to the applied membrane pressure during
a typical pressuremeter test. In that case, the initial shear modulus in the horizontal plane is given as

$$G_{hh} = \frac{1}{2} \left[ \frac{a\left(a^2 - b^2\right)(\mu_{'hh} + 2\mu_{''hh})^2(1 - \mu_{''hh} - 2\mu_{''hh}^2 - 1)}{a^2(1 + \mu_{''hh}) + b^2(1 - \mu_{''hh} - 2\mu_{''hh}^2)} \right] \left( \frac{dP}{du} \right)$$  \hspace{1cm} (21)

where:

$$\frac{dP}{du} = \text{the slope of the test-curve depicting the cell pressure as a function of the radial displacement of the membrane.}$$

If one considers instead the unload-reload portion of the test-curve (for a better assessment of the slope) then the radius of the membrane is $a + u$ (Warren 1982) and

$$G_{hh} = \frac{1}{2} \left[ \frac{(a + u)((a + u)^2 - b^2)(\mu_{''hh} + 2\mu_{''hh}^2 - 1)}{(a + u)^2(1 + \mu_{''hh}) + b^2(1 - \mu_{''hh} - 2\mu_{''hh}^2)} \right] \left( \frac{dP}{du} \right)$$  \hspace{1cm} (22)

This modulus is also obtained via the expression for the volumetric strain yielding

$$G_{hh} = \left[ \frac{(a + u)^2(1 - \mu_{''hh} - 2\mu_{''hh}^2)}{(a + u)^2(1 + \mu_{''hh}) + b^2(1 - \mu_{''hh} - 2\mu_{''hh}^2)} \right] \left( \frac{dP}{dv} \right)$$  \hspace{1cm} (23)

In pressuremeter testing, two independent measurements are taken, that of cavity pressure versus cell volume or that of cavity pressure versus radial displacement of the cell. Consequently, the two previous equations provide two independent paths in the determination of the elastic anisotropic moduli of confined pressuremeter tests.

From the above equations, one deduces that for a given stress reversal, there are a total of four unknown elastic parameters to be determined. Hence, if the pressuremeter test involves four or more stress reversals, there results a system of equations with four unknowns. The system spanned by Eq 22 and/or Eq 23 is rewritten in the form

$$G_{hh} = f_1(\mu_{hh}, \mu_{hv}, \mu_{'hh})$$  \hspace{1cm} (24)

$$\mu_{hh} = f_2(G_{hh}, \mu_{hv}, \mu_{'hh})$$  \hspace{1cm} (25)

$$\mu_{hv} = f_3(G_{hh}, \mu_{hv}, \mu_{'hv})$$  \hspace{1cm} (26)

$$\mu_{'hv} = f_4(G_{hh}, \mu_{hv}, \mu_{'hv})$$  \hspace{1cm} (27)

where:

$f$ = the iteration functions.
Specifically, they are given as

$$f_1 = \left[ \frac{(a + u)^2(1 - \mu_{hh} - 2\mu_{hh}^2)}{(a + u)^2(1 + \mu_{hh}) + b^2(1 - \mu_{hh} - 2\mu_{hh}^2)} \right] \left( \frac{dP}{du} \right)$$  \hspace{1cm} (28)

$$f_2 = \frac{(a + u)((a + u)^2 - b^2)\left(2\mu_{hv}, \mu_{'hv} - 1\right)}{2G_{hh}((a + u)^2 - b^2) - (a + u)((a + u)^2 - b^2)} \left( \frac{dP}{du} \right) - 2G_{hh}((a + u)^2 + b^2(1 - 2\mu_{hv}, \mu_{'hv}))$$  \hspace{1cm} (29)

$$f_3 = \frac{1}{2} \left[ (1 - \mu_{hh})b^2 + (1 + \mu_{hh})(a + u)G_{hh} + (\mu_{'hh} - 1)(a + u)^2 \right] \left( \frac{dP}{dv} \right)$$  \hspace{1cm} (30)

and

$$f_4 = \frac{(a + u)((a + u)^2 - b^2)(\mu_{hv}^2 - 1)}{-4G_{hh}b^2 - 2(a + u)((a + u)^2 - b^2) \left( \frac{dP}{du} \right) \mu_{hv}^2}$$  \hspace{1cm} (31)

A fixed-point iteration scheme is, therefore, utilized to solve for the cross-anisotropic elastic parameters representing the unknowns. The process begins with an initial estimate of the solution, which by way of iteration, is continuously improved until the desired accuracy is achieved. The estimated relative error is calculated for each of the elastic variables, and the iterations are stopped when the largest relative error is smaller than a specified value.
It must be emphasized that convergence of the method depends on the form of the iteration functions. In this particular study, the set of four equations produced by Eq 21 will converge under the following sufficient (but not necessary) conditions:

1. The iteration functions \( f_1, f_2, f_3, \) and \( f_4 \) and the partial derivatives of the iteration functions \( \frac{\partial f_1}{\partial \mu_{sh}} \), \( \frac{\partial f_2}{\partial \mu_{sh}} \), \( \frac{\partial f_3}{\partial \mu_{sh}} \), \( \frac{\partial f_4}{\partial \mu_{sh}} \), \( \frac{\partial f_1}{\partial \mu_{hh}} \), \( \frac{\partial f_2}{\partial \mu_{hh}} \), \( \frac{\partial f_3}{\partial \mu_{hh}} \), \( \frac{\partial f_4}{\partial \mu_{hh}} \), \( \frac{\partial f_1}{\partial \mu_{hv}} \), \( \frac{\partial f_2}{\partial \mu_{hv}} \), \( \frac{\partial f_3}{\partial \mu_{hv}} \), \( \frac{\partial f_4}{\partial \mu_{hv}} \) are continuous in the neighborhood of the solution.

2. \( \left| \frac{\partial f_1}{\partial \mu_{sh}} \right| + \left| \frac{\partial f_2}{\partial \mu_{sh}} \right| + \left| \frac{\partial f_3}{\partial \mu_{sh}} \right| \leq 1, \)
   \( \left| \frac{\partial f_1}{\partial \mu_{hh}} \right| + \left| \frac{\partial f_2}{\partial \mu_{hh}} \right| + \left| \frac{\partial f_3}{\partial \mu_{hh}} \right| \leq 1, \)
   \( \left| \frac{\partial f_1}{\partial \mu_{hv}} \right| + \left| \frac{\partial f_2}{\partial \mu_{hv}} \right| + \left| \frac{\partial f_3}{\partial \mu_{hv}} \right| \leq 1, \)

3. The initial values of \( \mu_{sh}, \mu_{hh}, \mu_{hv}, \mu_{ss}, \) and \( \mu_{hh} \) are sufficiently close to the solution.

As previously mentioned, the results of the confined laboratory pressuremeter tests are shown in Table 1. In this table \( (dP/dr) \equiv (dP/du) \) (vertical tests) is the slope of the test-curve depicting the cell pressure as a function of the radial displacement. Also shown is the back calculated cross-anisotropic shear moduli \( G_{hh} \) using Warren’s (1982) semi-infinite theory. Although this theory is strictly not applicable in the present context, it does provide an order of magnitude on the value of the anisotropic shear modulus one should expect from the confined tests. In fact, both the semi-infinite and confined theory yield very low radial strains further removed from the cavity. This even more so in the present tests because the radial displacements were kept extremely low in order to remain in the elastic domain.

Results of the back calculated cross-anisotropic moduli with the present confined theory are shown in Table 2. These results were obtained for relative strain amplitudes of the unload-reload cycle varying between 0.0041 and 0.0076 (Figs. 8 to 15). Based on the measured slopes of the unload-reload cycles given in Table 1, the cross-anisotropic elastic moduli \( G_{hh}, \mu_{hh}, \mu_{hh}', \) and \( \mu_{hv} \) are estimated as previously described. The elastic parameter \( G_{hh} \) is obtained using a relationship proposed by Gazetas (1981) and will be explained in the next section of this paper.

From the test results (Figs. 8–15) and Table 2, one deduces that the cross-anisotropic shear modulus \( G_{hh} \) is very dependent

### Table 1: Results of confined pressuremeter tests.

<table>
<thead>
<tr>
<th>Test Setup</th>
<th>Material</th>
<th>( \frac{dP}{dr} ) (MPa/m)</th>
<th>( \frac{dP}{du} ) (MPa/m³)</th>
<th>( G_{hh} ) Anisotropic Semi-Infinite Theory (Warren 1982) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>MG112 sand</td>
<td>939</td>
<td>35,000</td>
<td>8.6305</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2608</td>
<td>90,000</td>
<td>24.3817</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4036</td>
<td>138,000</td>
<td>38.0369</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5824</td>
<td>212,000</td>
<td>55.3210</td>
</tr>
<tr>
<td>Horizontal</td>
<td>MG112 sand</td>
<td>933</td>
<td>34,000</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1295</td>
<td>47,000</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3943</td>
<td>143,000</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5823</td>
<td>201,000</td>
<td>—</td>
</tr>
</tbody>
</table>

As a result of Eq 15, the Poisson’s ratio of strain in the vertical direction caused by strain in the horizontal direction is bracketed by

\[
-\sqrt{n \frac{1 - \mu_{hh}'}{2n}} \leq \mu_{hv} \leq \sqrt{n \frac{1 - \mu_{hh}'}{2n}}
\]
on the stress level developed during a given test. For a given material, the shear modulus increases with the stress level. The Poisson’s ratios, on the other hand, were less sensitive to the stress level. One should, however, be cautious with this last observation because the Poisson ratio is more prone to numerical inconstancies (round off as well as truncation errors) due to its low value.

The results in Table 2 also show that values of the cross-anisotropic modulus $G_{hh}$ obtained for MG20 gravel are generally higher than those obtained for MG112 sand. Furthermore, the magnitude of these values are confirmed by the range of values obtained with Warren’s (1982) theory (Table 1) although, as has been mentioned, this theory is not strictly applicable in the present context.

By comparing the Poisson ratios for the two materials, the difference is less obvious. It is difficult to explain this last observation at the present time without additional tests. However, one plausible contribution is certainly linked to the kinematic constraints imposed by the confinement of the material in the rigid mold.

The stage is now set for incorporating the obtained cross-anisotropic moduli in finite element simulations of both vertical and horizontal pressuremeter tests. This will assess whether or not the magnitude of the moduli are in the correct range.

### Table 2

<table>
<thead>
<tr>
<th>Material</th>
<th>$G_{hh}$ (Anisotropic Confined Theory Present Study (Eq 21) (MPa))</th>
<th>$\mu_{hh}$</th>
<th>$\mu_{vh}$</th>
<th>$\mu_{hv}$</th>
<th>$G_{vh}$ (Gazetas 1981) (Eq 34) (MPa)</th>
<th>Relative Strain Amplitude of Unload-Reload cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>MG20 gravel</td>
<td>9.8606, 20.1131, 41.6464, 85.4867</td>
<td>0.31, 0.29, 0.3, 0.29</td>
<td>0.34, 0.35, 0.34, 0.35</td>
<td>0.28, 0.27, 0.27, 0.28</td>
<td>10.8652, 22.1623, 45.8894, 94.1962</td>
<td>0.0073, 0.0063, 0.0046, 0.0006</td>
</tr>
<tr>
<td>MG112 sand</td>
<td>8.1805, 23.0679, 35.9554, 52.2477</td>
<td>0.3, 0.3, 0.3, 0.3</td>
<td>0.36, 0.35, 0.37, 0.37</td>
<td>0.28, 0.27, 0.28</td>
<td>9.0996, 25.6597, 39.9553</td>
<td>0.0076, 0.0058, 0.0049</td>
</tr>
</tbody>
</table>

The finite element study was carried out in 3D because axial symmetry prevailed only for the vertical tests. Figs. 16 and 17 show typical meshes, respectively, for the vertical and horizontal tests. A total of 92,574 tetrahedral elements were used in the simulations, which translates to a system with 50,271 df.

As for the boundary conditions, the mesh was constrained along all normal surface directions in addition to being restrained from movement at its base. The cavity pressure was applied in increments on the internal boundary of the mesh following the actual measured values of the laboratory tests.

The analytic theory for the confined pressuremeter tests (Theoretical Evaluation of the Cross-Anisotropic Elastic Moduli Based on a Confined Pressuremeter Test section) was used to back-calculate four $(G_{hh}, \mu_{hh}, \mu_{vh}, \mu_{hv})$ of the five independent moduli essentially in applying a cavity pressure either along the vertical direction or along the horizontal direction. Only the vertical pressuremeter tests were used in the back-calculation of the elastic parameters, whereas both the vertical and horizontal pressuremeter tests were used in assessing the parameters via finite element analysis.

### Finite Element Assessment of Anisotropic Elastic Parameters Obtained in Confined Pressuremeter Tests

A considerable amount of literature has been devoted to the finite element method (FEM). The mathematical approach to the FEM is general (Foriero et al. 2005; Foriero and Ladanyi 1995) and offers a wider applicability than that based on energy methods. The commercial finite element program COMSOL follows this approach and was chosen for the present analysis.

As previously mentioned, the confined pressuremeter tests were conducted via two different test set-ups, which consisted
parameters that describe completely the behavior of a homogeneous linearly elastic cross-anisotropic soil. For the finite element simulations, a good estimate of the fifth parameter $G_{vh}$ was obtained in Gazetas (1981). In that article, a constraint relationship for $G_{vh}$ proposed by Carrier (1946) is given as

$$G_{t}h = \frac{d_{11}d_{33} - d_{13}^2}{d_{11} + 2d_{13} + d_{33}}$$

(34)

where:

$$d_{11} = (E_{h}/a)(1 - n\mu_{vh}^2),$$

$$d_{12} = (E_{h}/a)(n\mu_{vh}^2 + \mu_{hh}),$$

$$d_{13} = (E_{h}/a)\mu_{vh}(1 + \mu_{hh}),$$

$$d_{33} = (E_{h}/a)(1 - \mu_{hh}^2),$$

$$n = (E_{v}/E_{h}),$$

and

$$a = (1 + \mu_{hh})(1 - \mu_{hh} - 2n\mu_{vh}^2).$$

The value of $G_{vh}$ obtained from the previous equation via the back-calculated values shown in Table 2, along with the values themselves, provided the necessary constitutive linear elastic cross-anisotropic elastic parameters for the finite element simulations. In COMSOL, one must specify the elasticity (stiffness) matrix, which is the inverse of the compliance matrix (obtained via Eqs 2–4). For a cross-anisotropic soil, the elasticity matrix in standard notation is given as

$$D = \begin{bmatrix}
\frac{E_{h}(\mu_{vh}^2\mu_{vh}^2 - 1)}{2\mu_{vh}(\mu_{vh}^2 + \mu_{hh}^2 - 1)(\mu_{hh}^2 + 1)} & -\frac{E_{h}(\mu_{vh}^2\mu_{vh}^2 + \mu_{lh}^2)}{(2\mu_{vh}(\mu_{vh}^2 + \mu_{hh}^2 - 1)(\mu_{hh}^2 + 1)} & -\frac{E_{h}(\mu_{vh}^2\mu_{vh}^2 + \mu_{hh}^2)}{(2\mu_{vh}(\mu_{vh}^2 + \mu_{hh}^2 - 1)(\mu_{hh}^2 + 1)}
0 & 0 & 0
0 & 0 & 0
0 & 0 & 0
\end{bmatrix}$$

(35)

The finite element results were generated by transforming the Cartesian stress and strain components into cylindrical components via the transformation matrix given by

FIG. 17 Typical finite element mesh for modeling horizontal pressuremeter tests.

FIG. 18 Finite element simulation of the von Mises stress resulting from a cavity pressure of 473 kPa in MG20 gravel.

**von Mises stress (N/m²)**
This yields for the cylindrical displacements in terms of the Cartesian displacements

\[
\begin{align*}
 u_r &= u \cos(\theta) + v \sin(\theta) \\
 u_\theta &= -u \sin(\theta) + v \cos(\theta) \\
 u_z &= w
\end{align*}
\]  

while the stress components from Cartesian to cylindrical coordinates are given as

\[
\begin{align*}
\sigma_r &= \sigma_x \cos^2(\theta) + \sigma_y \sin^2(\theta) + 2 \tau_{xy} \cos(\theta) \sin(\theta) \\
\sigma_\theta &= \sigma_x \sin^2(\theta) + \sigma_y \cos^2(\theta) - 2 \tau_{xy} \cos(\theta) \sin(\theta) \\
\sigma_z &= \sigma_z \\
\tau_{r\theta} &= -\sigma_x \cos(\theta) \sin(\theta) + \sigma_y \cos(\theta) \sin(\theta) + \tau_{xy} (\cos^2(\theta) - \sin^2(\theta)) \\
\tau_{r\theta} &= \tau_{xy} \cos(\theta) - \tau_{xy} \sin(\theta) \\
\tau_{xz} &= \tau_{xy} \sin(\theta) + \tau_{xy} \cos(\theta)
\end{align*}
\]  

Typical finite element results of the von Mises and radial stress for simulations in MG20 gravel are shown, respectively, in Figs. 18 and 19. These results were obtained at a pressure of 420 kPa over a cylindrical vertical cavity. Clearly, simulations demonstrate smoothness of the resulting stress field in the vicinity of the expanded cavity. This conforms with the expected and observed behavior during the tests. One also discerns that the stress field is localized because the stress produces infinitesimal strains. For the same material, Fig. 20 illustrates FEM results compared with the test results for the radial strain versus the cavity pressure. Agreement between the measured and finite element results increase as the magnitude of the moduli increase.

The previous observations could be carried over to the case of MG112 sand. Once again, when comparing FEM results with the test results, agreement between the measured and finite element results are analogous to the foregoing case. The difference is exclusively related to the magnitude of the cavity stresses, which are generally lower in the case of MG112 sand (Fig. 21).
As previously mentioned, only the results of the vertical pressuremeter tests were used to determine the cross-anisotropic elastic moduli (see Table 1). In order to verify the performance of the moduli under a different loading condition, a series of horizontal pressuremeter tests were also conducted in the same confining mold as that of the vertical tests.

In that case, typical finite element results of simulations in MG112 sand at a cavity pressure of 382 kPa were examined. The von Mises stress and the $y$-component of the stress tensor are shown respectively in Figs. 22 and 23. The stress field being continuous and localized in the vicinity of the cavity confirms once again that infinitesimal strains are predominant. In Fig. 24, FEM results are compared with the test results for the horizontal radial strain versus the horizontal cavity pressure. A similar conclusion is reached in that agreement between the measured and finite element results increase as the magnitude of the moduli increase.

**FIG. 21**
Finite element versus test results for vertical pressuremeter tests in MG112 sand.

**FIG. 22**
Finite element simulation of the von Mises stress resulting from a horizontal cavity pressure of 382 kPa in MG112 sand.

**FIG. 23**
Finite element simulation of the $y$ component of the stress tensor for a horizontal cavity pressure of 382 kPa in MG112 sand.
Finally, results of simulations in MG20 gravel are shown in Fig. 25 and reconfirm this last observation. Consequently, it is quite evident from this study that the cross-anisotropic moduli are strongly dependent on the level of stress. This fact is also confirmed in the articles of Foriero et al. (2014) and Puzrin and Burland (1998). Specifically, in those studies the cross-anisotropic elastic moduli are bounded by an inclined elliptical locus functionally dependent on the coupling between volumetric $p'$ and deviatoric $q$ components of stress.

Conclusions

The present study considered the determination of the cross-anisotropic elastic moduli of two base course materials (MG112 sand and MG20 gravel) via confined PENCEL pressuremeter tests. A special mold was designed in order to confine the base course material and to allow for pressuremeter testing in the vertical as well as horizontal directions. The vertical pressuremeter tests provided the necessary data for the determination of the cross-anisotropic moduli, while both the vertical and horizontal test results provided data for the assessment of the moduli.

A theoretical model was developed in order to simulate the expansion of a vertical cylindrical cavity in a confined mold. In this interpretation of the pressuremeter test, it is assumed that the pressuremeter is inserted into a cross-anisotropic medium with no disturbance of the surrounding soil. The model gives a relationship between the applied pressure and deformation of the soil in terms of the cross-anisotropic moduli.

Subsequently, this theoretical model was utilized in a numerical procedure whose objective was to back-calculate four of the five independent cross-anisotropic moduli, $G_{hh}$, $G_{ll}$, $G_{hv}$. 

**FIG. 24**

Finite element versus test results for horizontal pressuremeter tests in MG112 sand.

**FIG. 25**

Finite element versus test results for horizontal pressuremeter tests in MG20 gravel.
and $\mu'_{nh}$. One cannot overemphasize that only the vertical pressuremeter lab tests were used for the determination of the moduli.

Finally, in order to assess the performance of the obtained cross-anisotropic moduli, a finite element simulation of the various laboratory pressuremeter tests was conducted. Clearly, simulations demonstrated smoothness of the resulting stress field in the vicinity of the expanded cylindrical cavity for both vertical and horizontal pressures. This conforms with both the expected and observed behavior during the tests. Moreover, since the stress field is localized, this supports the assumption of elastic stresses producing infinitesimal strains on which the theoretical model rests. For all tests, agreement between the measured and finite element results increase as the magnitude of the moduli increase. This is irrespective of the material that is considered. The only apparent difference is exclusively related to the magnitude of the cavity stresses, which are generally lower for MG112 sand when compared to MG20 gravel.

More tests are required in order to gain confidence in the determination of the cross-anisotropic pressuremeter moduli using the confined mold. However the present tests confirm that one should consider the existence of cross-anisotropy as a rule, rather than as the exception.

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References