Using Dynamic Bayesian Networks to solve a dynamic reliability problem

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1 INTRODUCTION

In the domain of risk assessment, two approaches can be leaded in parallel and complete each other. The deterministic approach is mainly used to dimension a system and study the kinetics of the related physical phenomena. The probabilistic approaches assess the residual risk as the dependability of the system. In many applications, the dependability of a system may depend significantly on environmental variables. These physical phenomena are often considered through pessimistic assumptions. Consequently, the assessment of the dependability is often insensitive to much operating conditions variations. However, deterministic behaviour of environmental variables may be known. This knowledge can be taken into account to improve both the relevance of the modeling and the accuracy of the probabilistic assessment. Unfortunately, mixing these two points of view in an industrial context is a difficult task, particularly in the hydraulic domain.

Such systems are often referenced in the literature as forming part of the dynamic reliability framework. These systems are called hybrid systems. The evolution of hybrid systems is a combination between discrete stochastic events on the one hand and continuous or transitional deterministic phenomena on the other hand.

Mathematically, hybrid systems are generally represented by Piecewise Deterministic Markov Processes (PDMP) (Davis 1984). Using the PDMP framework to study the reliability of complex systems requires a two-stage procedure. Firstly, a description formalism is used to represent the various interactions of the system components. Secondly, quantification methods are needed to evaluate indicators related to the system dependability. Thus a challenge is to judiciously select both description and quantification methods regarding the characteristics of systems, the expected indicators and the level of quality required for the probability risk assessment.

1.1 State of the art

Some criteria are defined to select a description method as its clearness, ability to grab all the system characteristics of interest as complex interactions, whether a graphical representation can be used or not, the existence of software support tools, flexibility regarding the number of system configurations or environmental variables or limiting assumptions.

First the analytical methods consist in manually writing and solving the differential equations which govern the system evolution. Cocozza-Thivent et al. (2006) and Zhang et al. (2008) described any kind of interactions between components and environmental variables by solving Chapman-Kolmogorov equations, with or without the help of numerical algorithms. These methods allow to describe any small-sized systems. Moreover some extensions exist to adapt Petri Nets to hybrid systems as stochastic Petri Nets (Balbo et al. 1995, Dutuit et al. 1997, Chabot 1998, Dejean et al. 2005, Medjoudj and Yim 2007) or linear logic approach (Sadou and Demmou 2009). Recently, Perez Castaneda (2009) used a different view by implementing an Hybrid Stochastic Automaton (HSA). The Bayesian Network (BN) (Boudali and Dugan 2005) approach and its temporal extension called Dynamic Bayesian Network (DBN) (Murphy 2002) is an other relevant solution to describe hybrid systems. Other approaches based on modeling languages such as FIGARO or Altarica allow to describe the characteristics of important-sized systems by a set of logic rules (Ignizio 1991, Bouissou 2005). Finally, methods relying on a dynamic extension of the formalism of event trees have also been developed and tool provided. In (Labeau et al. 2000), the authors propose a review of such approaches describing some extensions of Discrete Dynamic Event Tree (DDET), Event Sequences Diagram (ESD) (Swaminathan and Smidts 1999) or Dynamic Flowgraph Methodology (DFM) (Aldemir et al. 2010).
Two classes of quantification methods can be considered: stochastic and deterministic algorithms. The Monte Carlo method (Gilks et al. 1996) is the most popular technique relying on stochastic simulations. The empirical probability of the occurrence of the top event is estimated by simulating many scenarios. The method is very flexible and can be applied to evaluate hybrid systems if they can be sampled (Marseguerra et al. 1998). Besides the accuracy of the result directly depends on the number of sampled scenarios. Consequently this can lead to intractable computations when rare events are considered even if extended algorithms exist to handle this case (Gilks et al. 1996, Marseguerra et al. 1998, Smidts and Devooght 1992).

Among deterministic methods, finite-volume algorithms (Eymard et al. 2000, Cocozza-Thivent et al. 2006) or Cell-to-Cell Mapping Technique (CCMT) (Aldemir 1987, Aldemir et al. 2010) are based on the discretization of the state space associated to the continuous variables. These approaches rely on an iterative resolution of Chapman-Kolmogorov equations (Cocozza-Thivent et al. 2006, Chiquet and Limnios 2008). A given finite-volume scheme allows to approximate the marginal probability distributions of the system state at any time. Besides sequence exploration methods (Bon et al. 1994) are a class of algorithms that estimates the probability of the top event by summing the probability of each sequence of events leading to it. Similarly, DDETs are associated with quantification methods which build the output of the dynamic risk analysis performed on the system, in terms of accident sequences and frequencies (Cojazzi 1996, Aldemir et al., Izquierdo et al. 1994). Moreover, the Stimulus-Driven Theory of Probabilistic Dynamics (SDTPD) make DDETs more efficient by coupling it with Monte Carlo simulations (Jourdain and Labeau 2011, Izquierdo and Labeau 2004).

1.2 Dynamic Bayesian Networks (DBN)

Jensen (1996) and Pearl (1988) have proved that using BNs is relevant to represent and analyse complex systems. For instance Boudali and Dugan (2005) show how to model the dependability of an industrial system by means of BNs. Weber and Jouffe (2003) studied the lifetime of a dynamic system represented by a Markov chain with Dynamic Bayesian Networks (DBNs). BNs are used to compactly represent graphically and quantitatively the joint distribution of a set of random variables. On the one hand the interest of using these tools stems from the powerful and intuitive graphical modeling capabilities. On the other hand the generic learning and inference tools allow respectively to fit the model parameters and to perform stochastic calculations.

In an industrial context, it is essential to get a representation of the system both trusty and readable. Furthermore, an well-expanded mathematical framework and existing software tools should lead to limit simplifying assumptions and decrease the computation time. Whereas it is not common in dynamic reliability yet, DBN formalism seems to connect these criteria. Thus it is chosen in the aim of representing a popular benchmark known as the heated tank system (Marseguerra and Zio 1996).

This article is divided into four sections. Section 2 briefly presents the PDMP theory and DBN formalism. In Section 3 the test case is described and some results are provided. Finally, some conclusions and perspectives are discussed in Section 4.

2 MATHEMATICAL FRAMEWORK

The continuous variables $X$ have deterministic trajectories between two jumps of the discrete variable $I$. Thus, the process $(I, X)$ is into the mathematical framework of piecewise deterministic stochastic processes. In lots of cases, the evolution of these systems validates the Markov assumption, which allows to use Piecewise Deterministic Markov Processes (PDMP) as an elaborated mathematical background (Davis 1984, Cocozza-Thivent et al. 2006).

2.1 Piecewise Deterministic Markov Process (PDMP)

A PDMP couples two components:

i. $(I_{t>0})$ is a discrete random variable with values in a finite or countable state space $\mathcal{E}$. $(I_{t>0})$ represents the system configuration;

ii. $(X_{t>0})$ is a continuous deterministic vector with values in $\mathbb{R}^d$ where $d$ is the number of continuous variables characterizing a physical phenomenon that affects the system.

The evolution of $X$, is governed by a differential equation depending on the system configuration $i \in \mathcal{E}$

$$\frac{dX_t}{dt} = v(i, X_t)$$ (1)

where $v$ is the velocity of $X$, when the system configuration is $i$. If the couple $(I_t, X_{t>0})$ is a PDMP, its marginal distribution is a solution measure to the equations of Chapman-Kolmogorov (Davis 1984).
2.2 Bayesian Networks (BN)

Bayesian Networks (BN) are mathematical tools based on the graph theory and the probability theory. They allow to represent intuitively and parsimoniously the joint distribution of random variables.

On the one hand, its structure is a directed acyclic graph, which is the qualitative part of the model. On the other hand, probability tables including the Conditional Probability Distributions (CPDs) of each random variable given its parents are the quantitative or probabilistic part of the model.

The factorization property is one of the main results associated with the BNs. The joint distribution of a random vector is given as the product of the CPDs associated to each variable

\[ P(X_1, \ldots, X_D) = \prod_{d=1}^{D} P(X_d | X_{pa_d}) \]  

where \( P(X_d | X_{pa_d}) \) is the CPD of variable \( X_d \) given its parents \( X_{pa_d} \).

When the graphical structure is known, learning algorithms have been developed to estimate the parameters of the CPDs according to possibly incomplete observations of the underlying random variables. Besides general inference algorithms have been built in order to perform efficient probabilistic computations into any BN-based models, exploiting the factorization property. Among the inference methods, the most generic is the bucket elimination algorithm (Dechter 1999).

If the considered random variables are time-indexed, the formalism of Dynamic Bayesian Networks (DBN) can be used (Murphy 2002) which is particularly the case within the domain of dynamic reliability (Donat et al. 2008).

3 APPLICATION

3.1 Description of the heated tank system

The heated tank system is a dynamic reliability problem well-known in the literature. It has been treated and solved first by Marseguerra (Marseguerra and Zio 1996) and Dutuit (Dutuit et al. 1997). Then it was chosen as application by some authors to experiment their methods and compare their results. Indeed, the heated tank system was described and quantified with analytical methods and Petri Nets (Zhang et al. 2008), colored Petri Nets (Sknourilova and Bris, Sknourilova and Bris 2008), ASH (Brinzei et al. 2009) or finite-volume scheme (Lair 2010). Except finite-volume approach, all these methods are based on Monte Carlo simulations.

The heated tank is a small but not trivial hybrid system. This system consists of a tank containing a fluid whose level is controlled by three hardware components: a main inlet pump \( P_1 \), a reserve inlet pump \( P_2 \) and an outlet valve \( V \). These components work independently and each of them has four possible states at time \( t \) denoted \( P_{1,t} \), \( P_{2,t} \) and \( V_t \) for \( P_1 \), \( P_2 \) and \( V \) respectively. These possible states are ON \((X_i = 1)\), OFF \((X_i = 2)\), stuck ON \((X_i = 3)\) and stuck OFF \((X_i = 4)\) for \( X = P_1, P_2, V \). Stuck ON and stuck OFF states are absorbing states in that it is assumed that the components cannot be repaired. If the component \( i \) is ON or stuck ON then its position \( O_{ij} = 1 \), else \( O_{ij} = 0 \). Nevertheless there are only six possible configurations \( C_i \) of the system in a physical sense, depending on the position of the components.

A thermal power source heats up the fluid and the failure rates \( \lambda_i \) of the components depend on the temperature. It is an inhomogeneous Poisson jumps process. Three possible top events are considered: dryout \((L < 4m)\), overflow \((L < 10m)\) and hot temperature \((\Theta \geq 100 ^\circ C)\). The objective is to calculate the cumulative probabilities of these events at time \( t \).

Control laws modify the state of the components to keep the fluid in a “correct functioning” region of level. Each time the level comes out the “correct functioning” region or near to it, until the next stochastic transition.

The dryout and overflow top events have been assumed to occur at \( L < 4m \) and \( L < 10m \) respectively. The two control thresholds have been set at \( L < 6m \) and \( L < 8m \). Then there are three possible regions of level \( RL \): \([4;6]\), \([6,8]\) and \([8,10]\). The hot temperature top event has been assumed to occur at \( \Theta \geq 100 ^\circ C \). Then two process variables are of interest: the level \( L \) and the temperature \( \Theta \), of the fluid in the tank at time \( t \). The following system of differential equations (3) is deduced from the mass and energy conservation laws:

\[
\begin{align*}
\frac{dL_t}{dt} &= \gamma_1 (O_{t-1}^1) - \gamma_2 (O_{t-1}^3) L_{t-1}^4 & 4 \leq L_t < 10 \\
\frac{d\Theta_t}{dt} &= \frac{\gamma_2 (O_{t-1}^3 - \gamma_3 (O_{t-1}) \Theta_{t-1})}{L_{t-1}} & 0 \leq \Theta_t < 100
\end{align*}
\]

where

\[
\begin{align*}
\gamma_1 (O_t) &= (O_t^1 + O_t^2 - O_t^3) G, \\
\gamma_2 (O_t) &= (O_t^4 + O_t^2) G \Theta_m + \omega, \\
\gamma_3 (O_t) &= (O_t^1 + O_t^2) G.
\end{align*}
\]
According to Zhang et al. (2008), $G = 1.5 \text{ m} \cdot \text{h}^{-1}$, $\theta_0 = 15 ^\circ \text{C}$ and $w = 23.88915 \text{ m} \cdot \text{C} \cdot \text{h}^{-1}$, with $O_t = [O_{c,1}, O_{c,2}, O_{c,3}]$.

Furthermore $\lambda_i(\Theta_t) = a(\Theta_t) \hat{\lambda}_i$, $i = 1, 2, 3$, with $\hat{\lambda}_1 = 2.2831 \cdot 10^{-3} \text{ h}^{-1}$, $\hat{\lambda}_2 = 2.8571 \cdot 10^{-3} \text{ h}^{-1}$, $\hat{\lambda}_3 = 1.5625 \cdot 10^{-3} \text{ h}^{-1}$ and

$$a(\Theta_t) = \frac{b_1 e^{h(\Theta_t - \theta_{wa})} + b_2 e^{-b_2(\Theta_t - \theta_{wa})}}{(b_1 + b_2)}$$ (7)

where $b_1 = 3.0295$, $b_2 = 0.7578$, $b_C = 0.05756$, $b_\theta = 0.2301$ and $\theta_{wa} = 20 ^\circ \text{C}$.

Finally, knowing that the components are unstuck, control laws are written

$$\begin{aligned}
\text{if } L_t < 6 & \text{ then } P_{1t} = 1, P_{2t} = 2, V_t = 1 \\
\text{if } 6 \leq L_t < 8 & \text{ then } P_{1t} = 1, P_{2t} = 1, V_t = 2. \\
\text{if } 8 \leq L_t & \text{ then } P_{1t} = 2, P_{2t} = 2, V_t = 1 
\end{aligned}$$

3.2 Graphical modeling

The DBN describing the dependencies between the different variables of interest of the heated tank is shown in Figure ??.

The graphical structure of the DBN in Figure 1 describes the following dependency relations:

1. The level $L_t$ is updated: $L_{t+1}$ depends on the last level $L_t$ and the last configuration $C_t$.
2. The region of level $RL_{t+1}$ is updated: $RL_{t+1}$ depends on the present level $L_{t+1}$.
3. The temperature $\Theta_t$ is updated: $\Theta_{t+1}$ depends on the last temperature $\Theta_t$, the last configuration $C_t$ and the last level $L_t$.
4. The state of $P1$ is updated: $P1_{t+1}$ depends on the last state $P1_t$, the present region of level $RL_{t+1}$ (according to the control laws) and the last temperature $\Theta_t$ (because the failure rate depends on the temperature).
5. The state of $P2$ is updated: $P2_{t+1}$ depends on the last state $P2_t$, the present region of level $RL_{t+1}$ and the last temperature $\Theta_t$.
6. The state of $V$ is updated: $V_{t+1}$ depends on the last state $V_t$, the present region of level $RL_{t+1}$ and the last temperature $\Theta_t$.
7. The configuration $C_t$ is updated: $C_{t+1}$ depends on the present states of the components $P1_{t+1}$, $P2_{t+1}$ and $V_{t+1}$.

With the aim of calculating the probabilities of top events $\{\Theta_{t} \geq 100\}, \{H < 4\}$ and $\{H \geq 10\}$, the joint distribution of the process $P(P1_t, P2_t, V_t, L_t, \Theta_t)$ should be calculated.

3.3 Quantification method

Our quantification approach is based on the bucket elimination in association with the BDN formalism (Dechter 1999). At first, discretization steps of continuous variables should be defined. In our case $\delta_t = 1h$, $\delta_l = 0.5m$ and $\delta_\theta = 1 ^\circ \text{C}$ are chosen. Then conditional probability distributions are referenced in probability tables. Only CPDs of $P1_t$, $P2_t$ and $V_t$ are not deterministic. The joint distribution of the process is calculated from a recursive algorithm based on the factorization property. This algorithm computes $P(P1_{t+1}, P2_{t+1}, V_{t+1}, L_{t+1}, \Theta_t)$ from $P(P1_t, P2_t, V_t, L_t, \Theta_t)$ and the conditional distribution of $P1_t, P2_t, V_t, L_t, \Theta_t, RL_t$ and $C_t$.

**Step 0(a).**

$$P(C_t, V_{t+1}/P1_t, P2_t, V_t, RL_{t+1}, \Theta_t) = P(C_t/P1_t, P2_t, V_t) \times P(V_{t+1}/V_t, RL_{t+1}, \Theta_t).$$ (8)

![Figure 1. Graphical representation of the heated tank system.](image-url)
Step 0(b).

\[
P(L_{t+1}, \Theta_{t+1}/C_t, L_t, \Theta_t) = P(L_{t+1}/C_t, L_t) \times P(\Theta_{t+1}/C_t, L_t, \Theta_t).
\]  

(9)

Step 1.

\[
P(V_{t+1}, P_{1t}, P_{2t}, L_t, \Theta_t, C_t/RL_{t+1}) = \sum_{V_t} P(P_{1t}, P_{2t}, V_t, L_t, \Theta_t) \times P(C_t, V_{t+1}/P_{1t}, P_{2t}, V_t, RL_{t+1}, \Theta_t).
\]  

(10)

Step 2.

\[
P(P_{2t+1}, V_{t+1}, P_{1t}, L_t, \Theta_t, C_t/RL_{t+1}) = \sum_{P_{1t}} P(P_{2t+1}/P_{2t}, RL_{t+1}, \Theta_t) \times P(V_{t+1}, P_{1t}, P_{2t}, L_t, \Theta_t, C_t/RL_{t+1}).
\]  

(11)

Step 3.

\[
P(P_{1t+1}, P_{2t+1}, V_{t+1}, L_t, \Theta_t, C_t/RL_{t+1}) = \sum_{P_{1t}} P(P_{1t+1}/P_{1t}, RL_{t+1}, \Theta_t) \times P(P_{2t+1}, V_{t+1}, P_{1t}, L_t, \Theta_t, C_t/RL_{t+1}).
\]  

(12)

Step 4.

\[
P(P_{1t+1}, P_{2t+1}, V_{t+1}, L_t, \Theta_t, C_t/L_{t+1}) = \sum_{RL_{t+1}} P(RL_{t+1}/L_{t+1}) \times P(P_{1t+1}, P_{2t+1}, V_{t+1}, L_t, \Theta_t, C_t/RL_{t+1}).
\]  

(13)

Step 5.

\[
P(P_{1t+1}, P_{2t+1}, V_{t+1}, L_{t+1}, \Theta_{t+1}) = \sum_{C_t} \sum_{L_t} \sum_{\Theta_t} P(L_{t+1}, \Theta_{t+1}/C_t, L_t, \Theta_t) \times P(P_{1t+1}, P_{2t+1}, V_{t+1}, L_t, \Theta_t, C_t/L_{t+1}).
\]  

(14)

Then the value of \( P(P_{1t_{rel}}, P_{2t_{rel}}, V_{rel}, L_{rel}, \Theta_{rel}) \) is moved to \( P(P_{1t_{rel}}, P_{2t_{rel}}, V_{rel}, L_{rel}, \Theta_{rel}) \) and step 1 is repeated.

3.4 Results

At first, let note that the results significantly depend on the sequence in which variables are updated during inference. For example, \( P_{1t_{rel}}, P_{2t_{rel}} \) and \( V_{rel} \) may be updated with values of \( (L_t, \Theta_t) \) or \( (L_{rel}, \Theta_{rel}) \). This is a matter related to the numerical resolution which is based on discretization. In particular, this problem arises from the way of the differential equations are represented in the CPDs which is related to both the qualitative understanding of the system and the way of the discretization of the continuous equations is performed. On the one hand, the chosen sequence may reflect an assumption about the system working. This type of dependencies are not implicit in references, particularly for time-continuous methods. On the other hand, it could be a simple matter of adjusting the discretization step. In the worst case, this may be a stability problem of the approach. Because we plan to adopt this method in our future work, it is a question for us to study in depth. Here we chose the update sequence leading to closest results to those presented in the reference paper (Zhang et al. 2008). In future perspectives, we should make sure to have precise rules to describe the system. In parallel, work will be led to solve this order problem and refine discretization steps.

Cumulative probabilities of the top events are computed with the DBN approach. Then we confront our results with those obtained by Zhang et al. (2008) with Monte Carlo simulations. Our results are consistent in comparison with the literature. However, our computation times are very long due to a lack of optimization of our algorithm. Indeed time and continuous variables discretization leads to an important space complexity. To our knowledge, only Lair (2010) has already used this kind of discretization approach based on a finite-volume scheme that leads to reasonable computation times even with very small discretization steps. Unfortunately, the finite-volume scheme is application dependent and a rewriting of the quantification algorithm is needed in this case. On the other hand, it is worth noting that this type of benchmark problem has been solved within...
seconds with DDETs and Monte Carlo simulations (Cojazzi 1996). Numerical improvements of DBN based quantification method, particularly by exploiting the deterministic parts of the CPDs describing the behaviour of environmental variables, should provide computation times closed to the state of the art.

Our approach based on the use of DBNs to represent and quantify hybrid systems is original in that a deterministic inference method is applied, instead of the classic Monte Carlo method. If we optimized our algorithm, our approach might become competitive. Besides the DBN approach would be generalizable. Indeed current work shows it would be possible to automatically generate a graphical modeling of the DBN of a system, according to very precise rules. Once the system is described by a DBN, it is possible to modify the behaviour of the studied system without writing a new quantification algorithm. For example this can be useful to measure the effect of introducing a new component in the system.

4 CONCLUSION

The proposed method consists on the association of DBN formalism and a bucket elimination algorithm. This algorithm allows to describe and solve a small but not trivial system. With this approach, characteristics of the system are described readily in a DBN. Furthermore the results from the quantification of the system are satisfying in comparison with the literature. These encouraging results confirm that DBN are competitive tools for practical dynamic reliability problem. In future work we will make our quantification algorithm faster by optimizing calculations with sparse matrix in order to exploit the important deterministic parts of CPDs. Then, the objective will be to apply this approach to industrial-sized complex systems thanks to the powerful DBN modeling formalism.

REFERENCES


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